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Flexural strength of steel fibre-reinforced cement composites

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The paper considers two existing theories for the flexural strength of steel fibre-reinforced cement composites, which are based on the post-cracking strength concept and on the rule of mixtures, respectively. It is shown that the former theory has serious limitations because of the omission of the matrix strength contribution to the ultimate composite strength. The rule of mixtures is strictly valid for composites under a direct tensile state of stress and its extension to the flexural state of stress has not been theoretically verified. An expression for the flexural strength of steel fibre-reinforced cement composites has been derived, based on an assumed stress block and fundamental principles of flexural mechanics. The derived expression for flexural strength is shown to be valid to the experimental results of this investigation and to the flexural strength data available from previous research. In the experimental part of this investigation, the $v_f(l/d)$ ratio of fibres in a wide range of cement matrices was kept constant. Variations in composite strength were achieved by different mix proportions of the matrix and by long-term curing both under marine exposure and under laboratory curing. In the experimental results from previous research, however, changes in composite strength were caused by different $v_f(l/d)$ ratios of fibres.

1. Introduction

The mechanics of fibre reinforcement of brittle matrices in tension has been dealt with in detail by Aveston *et al.* [1, 2]. Such composites are characterized by three distinct modes of tensile failure. (a) The composite is "matrix controlled" and upon cracking of the brittle matrix, fibres are unable to sustain the transferred stress. Consequently, composite failure is instantaneous. This type of composite has little practical significance. (b) The composite is still "matrix controlled" but carries a decreasing load after matrix cracking as the fibres pull out from the cracked surface, final failure being by a single fracture. This type of failure is typical of some cement matrices reinforced with short, randomly orientated steel or organic fibres at relatively small volumes. (c) The composite is "fibre controlled" and after cracking of the brittle matrix, the fibres continue to carry an increasing tensile stress resulting in multiple cracking of the matrix. This behaviour is characteristic of cements reinforced with relatively large volume fractions of continuous fibres.

The transition from a single to multiple fracture of the matrix can be schematically represented as shown in Fig. 1. Line XY in Fig. 1 is valid when the composite is "matrix controlled" and line PQ represents "fibre controlled" composites in which fibres sustain increasing stress after matrix cracking until either their ultimate tensile stress is reached or fibre–matrix interfacial bond failure occurs. For brittle matrix composites, line XY is valid up to the critical volume fraction, v_{crit}, beyond which line PQ becomes valid.

2. Limitations of existing theories

It is now well established that the flexural load sustained by a concrete beam can be considerably increased by the inclusion of steel fibres whereas there is only a marginal increase in the uniaxial tensile strength of the material [3]. This is because in flexure an increase in bending moment can be accommodated by a shift of the neutral axis towards the compression surface as the tensile zone becomes inelastic at high stresses. A precise interpretation of the reinforcement mechanism in flexure presents considerable theoretical problems [2–5] but is, nevertheless, important because a majority of applications of fibre-reinforced cementitious materials involve flexural stresses. Several theoretical studies of flexural behaviour have been made in recent years but two of these approaches are considered in this paper [3, 6].

2.1. "Post-cracking" strength concept

A "post-cracking" flexural strength theory was derived by Hannant [3] on the basis of a fundamental assumption that at ultimate stress the matrix is fully cracked and, therefore, composite strength is purely a function of the pull-out resistance offered by the fibres bridging the cracks. No contribution of the matrix strength was, therefore, included in the composite flexural strength expression. Based on the generally recognized principles of ultimate flexural behaviour and the above assumptions, Hannant [3] derived the following expression for the fictitious or apparent modulus of rupture, σ_{cu}, of such composites as steel fibre-reinforced concrete (sfrc)

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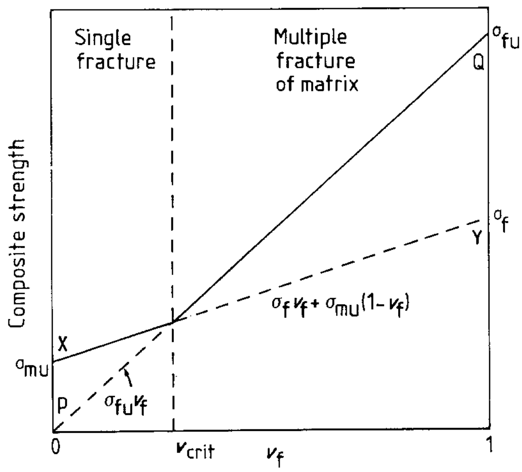

Figure 1 Relation between composite tensile strength and fibre volume for a brittle matrix reinforced with ductile fibres.

$$\sigma_{cu} = \tfrac{39}{32} v_f \tau \left(\frac{l}{d}\right) \quad (1)$$

where v_f is the volume fraction of fibres, τ is average fibre–matrix bond strength at ultimate load, l/d is the fibre aspect ratio.

The apparent modulus of rupture expression was derived by equating the ultimate moment of resistance derived from the assumed stress block at ultimate load to the moment of resistance obtained assuming a classical elastic stress block at ultimate load.

On the basis of this "post-cracking" strength concept, the following expression for the direct tensile strength, σ_{ct}, of composites such as sfrc was also derived [3]

$$\sigma_{ct} = \tfrac{1}{2} v_f \tau \left(\frac{l}{d}\right) \quad (2)$$

The expression for flexural strength (Equation 1) is inadequate for sfrc mixes which are commonly in use because the contribution of the matrix phase to strength has been neglected. This limitation can be clearly demonstrated if values of τ are calculated using Equation 1 for a wide range of flexural strength data on sfrc. The data on flexural strength, σ_{cu}, was obtained from Mangat [7] and from the present investigation, for a wide range of $v_f(l/d)$ ratios, types of steel fibres and mix proportions. The fibre volume fractions used in these mixes ranged between 0.5% and 2.5% which are typical of practical sfrc. These values of τ are plotted against $v_f(l/d)$ in Fig. 2 which shows that the value of τ increases from about $4\,\mathrm{N\,mm^{-2}}$ at a $v_f(l/d)$ ratio of 2.4 to a value of about $13\,\mathrm{N\,mm^{-2}}$ at a $v_f(l/d)$ ratio of 0.4. The clear trend of τ increasing with decreasing $v_f(l/d)$ ratio is due to the fact that the matrix strength component has not been included in the composite strength Equation 1. This leads to artificially large values of τ to compensate for the matrix strength contribution. In fact, the large τ values obtained at low $v_f(l/d)$ ratios are inconceivable because they are well in excess of the tensile or shear strength of cement matrices.

Values of τ were also calculated from Equation 2 using a range of direct tensile strength results on sfrc from Johnston and Coleman [8]. These τ values are plotted against $v_f(l/d)$ in Fig. 3. A sharp increase in τ with decreasing $v_f(l/d)$ is again evident which, as will be confirmed later, is because of the omission of the matrix strength component in Equation 2.

The implications of the above observations are that in the sfrc mixes represented above, $v_f \leqslant v_{crit}$ (see Fig. 1). As a result the composites are matrix controlled and simultaneous matrix cracking and composite failure are expected at ultimate load. The failure mode is due to the propagation of a single crack and there is no occurrence of multiple cracking. This is borne out by numerous observations of flexural tests on sfrc both in this investigation and by others [9].

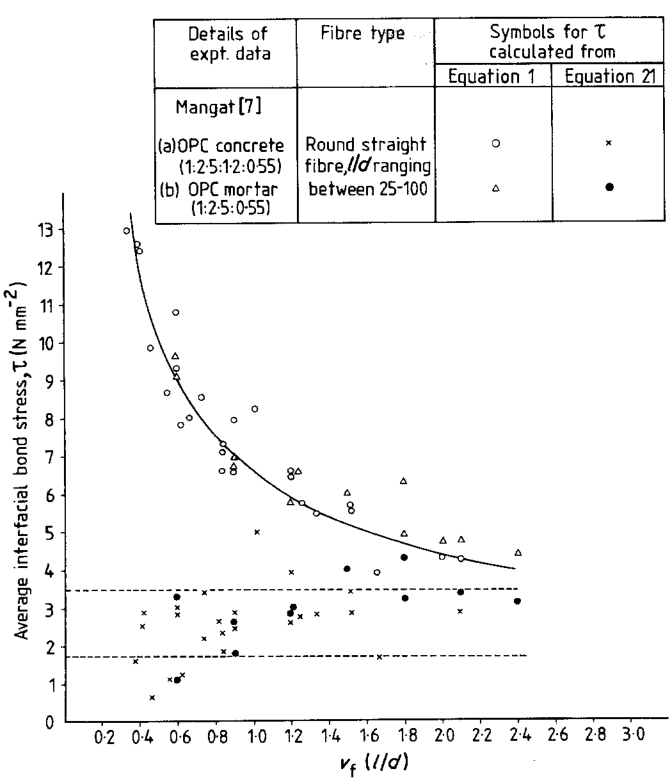

Figure 2 The average interfacial bond stress (τ) at ultimate flexural load as a function of $v_f(l/d)$.

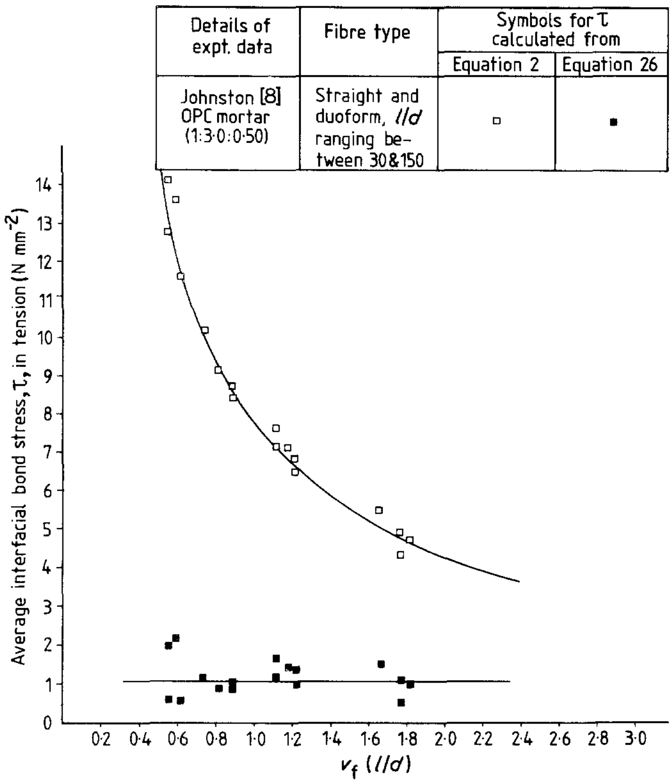

Figure 3 Average interfacial bond stress (τ) at ultimate tensile load as a function of $v_f(l/d)$.

2.2. Mechanics of composite materials approach

Using the law of mixtures as a basis, Swamy and Mangat [6] derived the following expression for the flexural strength of concrete reinforced with randomly oriented short steel fibres.

$$\sigma_c = A\sigma_m(1 - v_f) + 0.82\tau v_f(l/d) \qquad (3)$$

where σ and v represent stress and volume, respectively, and suffixes c, m and f represent composite, matrix and fibre, respectively.

By regression analysis of a wide range of flexural strength data, Swamy and Mangat [6] obtained the following equations:
first crack composite flexural strength

$$\sigma_{cf} = 0.843\sigma_m(1 - v_f) + 2.93v_f\frac{l}{d} \qquad (4)$$

ultimate composite flexural strength

$$\sigma_{cu} = 0.97\sigma_m(1 - v_f) + 3.41v_f\frac{l}{d} \qquad (5)$$

Although the above theory has shown very good correlation with experimental data, the validity of the law of mixtures to composites in flexure can be questioned at a fundamental level.

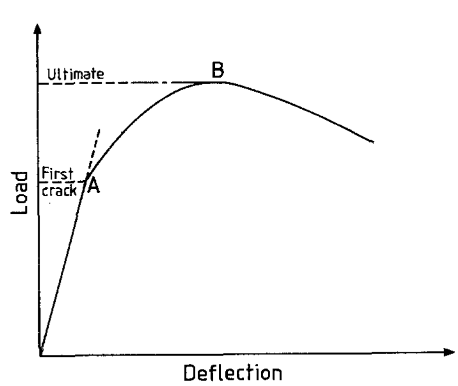

Figure 4 Typical load–deflection curve for steel fibre-reinforced concrete in flexure.

3. Theoretical analysis of flexural strength

3.1. Mode of failure

In order to derive the stress block for sfrc at ultimate (maximum) flexural stress (see Fig. 4), an appreciation of the load–deflection behaviour is important. Fig. 4 illustrates a typical load–deflection curve in flexure. The curve is linear up to point A which is commonly defined as the first crack stress. Further loading leads to a non-linear load–deflection curve represented by portion AB. At ultimate (maximum) load, simultaneous matrix cracking and composite fracture occur but sudden failure of the specimen is prevented because of the frictional resistance of fibres during pull-out.

Beyond the ultimate load, the composite carries a decreasing load as fibres offer resistance to pull-out through frictional shear bond. Because the friction shear bond of steel fibres in cement matrices is comparable to ultimate shear bond [10], the post-ultimate load capacity of the composites can be significant.

3.2. Stress block in flexure

The analysis of ultimate flexural strength is based on a simplified stress block shown in Fig. 5. The matrix tensile strength, σ_{mt}, is represented by $K\sigma_m$ where σ_m is the modulus of rupture of plain concrete calculated from elastic theory and K is an empirical constant [11]. σ_f is the average stress sustained by fibres in the tensile zone, which is dependent on the average ultimate bond stress, τ. The factor α is introduced to allow for the shift in neutral axis towards the compression zone at ultimate load.

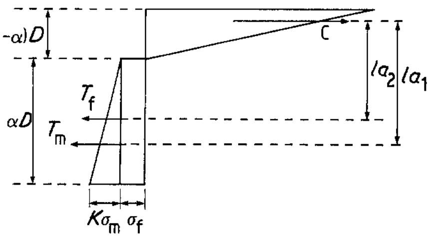

Figure 5 Simplified stress block for sfrc at ultimate load in flexure.

It is recognized that Fig. 5 may not represent precisely the ultimate stress distributions in the fibre and matrix phases, which are likely to be non-linear to some degree. However, as will be seen later, this does not alter the basic nature of the flexural strength equations derived. The stress distribution in the compression zone of Fig. 5 is assumed linear because the non-linearity of the load–deflection curve between points A and B in Fig. 4 is caused primarily by the inelasticity of the tensile zone and shifting of the neutral axis. The strains on the extreme compression face of specimens near failure were found to be of the order of 500 microstrain which is a small fraction of the ultimate compressive strain capacity of concrete which is about 3500 microstrain. Consequently, concrete in the compression zone can be assumed to be approximately elastic, resulting in linear stress distribution in Fig. 5.

3.3. Derivation of flexural strength equation

From Fig. 5, the moment of resistance (MR) about the line of action of the compressive force, C, is

$$\mathrm{MR} = T_m la_1 + T_f la_2 \tag{6}$$

where T_m and T_f are the tensile resistance forces mobilized by the matrix and fibres, respectively, and la_1 and la_2 are their respective lever arms.

T_f is dependent on the number of fibres per unit area, N, across a cross-section, the average bond stress τ and the average length of fibres bridging the plane of the failure crack. For a section of unit width, the area of fibres, A_f, bridging a cross-section in the tensile zone is given by

$$A_f = N\pi \frac{d^2}{4} D\alpha \tag{7}$$

where d is the fibre diameter. Hence,

$$T_f = \sigma_f N\pi \frac{d^2}{4} D\alpha \tag{8}$$

where σ_f is the average tensile stress in fibres at ultimate flexural load. σ_f is dependent on τ and the mean fibre length bridging the failure plane, which is assumed to be $l/4$ [2]. The average tensile force per fibre, t_f, at ultimate flexural load is then given by the expression:

$$t_f = \tau\pi d \frac{l}{4} \tag{9}$$

and

$$\sigma_f = \frac{t_f}{(\pi d^2/4)} \tag{10}$$

Therefore, from Equations 9 and 10

$$\sigma_f = \tau \frac{l}{d} \tag{11}$$

The number of fibres per unit area, N, in a composite reinforced with randomly oriented and uniformly distributed fibres can be shown to be [3]

$$N = \frac{\beta v_f}{(\pi d^2/4)} \tag{12}$$

where β is the fibre orientation factor.

Substituting from Equations 11 and 12 into Equation 8 gives

$$T_f = \tau \frac{l}{d} \beta v_f D\alpha \tag{13}$$

From the matrix stress block in Fig. 5

$$T_m = \frac{K\sigma_m}{2}(A_c - A_f) \tag{14}$$

where A_c is the gross cross-sectional area of the tensile zone, which equals αD.

Substituting from Equations 7 and 12 into Equation 14 gives

$$T_m = \frac{K\sigma_m}{2} D\alpha(1 - \beta v_f) \tag{15}$$

From Fig. 5, the lever arms la_1 and la_2 can be expressed as:

$$la_1 = \tfrac{2}{3}D \tag{16}$$

$$la_2 = \frac{D}{6}(4 - \alpha) \tag{17}$$

Hence substituting from Equations 13, 15, 16 and 17 into Equation 6 gives

$$\mathrm{MR} = \left(\frac{\alpha D^2}{3}\right) K\sigma_m(1 - \beta v_f) + \left[\alpha D^2 \frac{(4-\alpha)}{6}\right] \tau\beta v_f \frac{l}{d} \tag{18}$$

The flexural strength of sfrc is usually calculated using elastic theory based on a triangular stress block with the neutral axis coinciding with the centroidal axis of the section. This is a convenient simplification which, in fact, does not give the true tensile strength of the composite in flexure and can be referred to as the fictitious or apparent modulus of rupture [3]. This is because the increased area of the tensile stress block caused by the shift of neutral axis at ultimate load is not taken into account. Nevertheless this value of modulus of rupture is of great practical importance and is invariably used in engineering design and for quality control purposes.

An expression for the apparent modulus of rupture of sfrc can be derived by equating the moment of resistance based on the elastic stress block to the moment of resistance given by Equation 18. A triangular stress block based on elastic theory gives the following expression for moment of resistance

$$\mathrm{MR} = \frac{\sigma_{cu} D^2}{6} \tag{19}$$

where σ_{cu} is the modulus of rupture of the composite.

Equating the ultimate moment of resistance values given by Equations 18 and 19 gives

$$\sigma_{cu} = (2\alpha)K\sigma_m(1 - \beta v_f) + \alpha(4 - \alpha)\tau\beta v_f \frac{l}{d} \quad (20)$$

It should be noted that any variations in assumptions with respect to the assumed stress block in Fig. 5 will not affect the basic nature of Equation 20 but will only alter the values of the constants.

It has been stated [3] that for sfrc, the neutral axis may only be $0.8D$ from the tensile face at ultimate flexural load. Results from this investigation, which are presented in Fig. 6 show the relationship between depth of neutral axis and crack width at the tension face. Data for specimens reinforced with 1.7 vol % melt extract steel fibres having an aspect ratio of 60 are given. Neutral axis depths at small crack widths correspond to the maximum stress sustained by the specimens. Higher crack widths correspond to the falling branch of the load–deflection curve (Fig. 4). The results at the small crack widths show considerable scatter and an average value for neutral axis depth of $0.75D$ is assumed. This assumption was also made by Hannant [3] in his flexural analysis. Hence substituting the value of 0.75 for α in Equation 20 gives

$$\sigma_{cu} = 1.5K\sigma_m(1 - \beta v_f) + 2.44\tau\beta v_f \frac{l}{d} \quad (21)$$

Equation 21 is similar to the expression derived by Swamy and Mangat [6] using the law of mixtures as a basis. This gives the modulus of rupture of the composite, σ_c, as given by Equation 3. A regression analysis on a wide range of experimental data simplified Equation 3 to Equation 5. A comparison of Equations 5 and 21 indicates that factor $1.5K$ in Equation 21 should equal 0.97 which gives a value for K of 0.65. The empirical constant K relates the direct tensile strength, σ_{mt}, and the modulus of rupture, σ_m, of concrete matrices in the form

$$\sigma_{mt} = K\sigma_m \quad (22)$$

Values of K given in the literature [11], for different mix proportions, range between 0.63 and 0.83. A possible value of 0.65 in Equation 21 is clearly acceptable.

The second term on the right-hand side of Equations 21 and 3, the parameter $\tau v_f(l/d)$, is common to both equations. The constant 2.44β in Equation 21, however, does not simplify to the value of 0.82 (equation 3) if, as in the derivation of Equation 3, the orientation factor β is assumed as 0.41. But this constant would equal 0.82 if the value of α in the analyses was assumed to be 0.5 and if the fibre stress distribution in the tensile zone in Fig. 5 was assumed as triangular. This implies that the law of mixtures approach adopted by Swamy and Mangat [6] is a special case of the general approach adopted in this paper.

The term $(1 - \beta v_f)$ in Equation 21 represents the volume of the matrix, v_m, in the composite reinforced with an equivalent volume βv_f of aligned fibres. Swamy and Mangat [6] have omitted the term β when representing the volume of matrix in their Equation 3. In practical sfrc composites, however, their omission is not significant because maximum values of v_f are relatively small and the errors induced in using either $(1 - v_f)$ or $(1 - \beta v_f)$ are insignificant. For the sake of generality, however, Equation 21 is appropriate. Based on Equation 21, a regression analysis of the extensive range of flexural strength data which were used by Swamy and Mangat [6], yield the following design expression

$$\sigma_{cu} = 0.97\sigma_m(1 - \beta v_f) + 3.41 v_f \frac{l}{d} \quad (23)$$

which is theoretically more accurate than the design expression in Equation 5.

Depth of specimens, D=100mm
Mix proportions: 0·26:0·74:1·51:0·84:0·40

Fibre type	Melt extract (ME)	Low carbon steel (MS)
Symbol	△	▲
$v_f(l/d)$	100	100
Mix	B_{ME}	B_{MS}

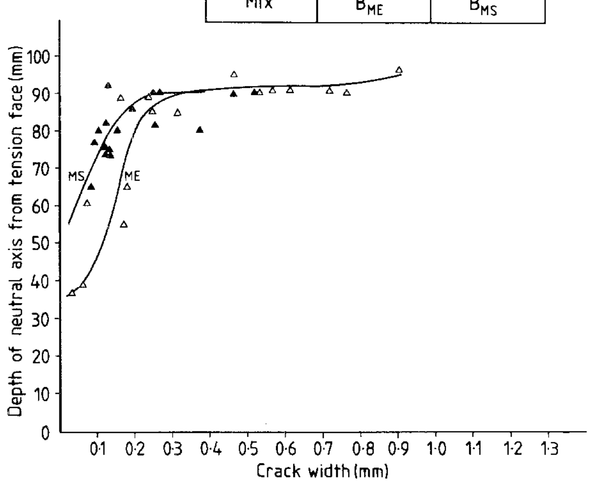

Figure 6 The shift of neutral axis from the tension surface of prism specimens tested in flexure.

4. Experimental investigation

In most of the existing data on sfrc [3, 6, 9], the variations in composite strength, σ_{cu}, were achieved primarily by adding different $v_f(l/d)$ ratios of steel fibres in cement matrices. In the experimental part of this investigation, the $v_f(l/d)$ ratio of different types of steel fibres used was kept practically constant. Variations in strength, σ_{cu}, were achieved by using different test ages, mix proportions of the matrix and different curing conditions. The validity of the flexural strength expressions to these data is examined in this section.

4.1. Details of tests

Two concrete mixes were used in this investigation. The first mix (Mix A) was of proportions by weight of ordinary Portland Cement (OPC) : fine aggregate : coarse aggregate of 1 : 1.15 : 0.86 with a water/cement ratio of 0.4. The second mix (Mix B) incorporated pulverized fuel ash (pfa) and was of proportions by weight of pfa : OPC : fine aggregate : coarse aggregate of 0.26 : 0.74 : 1.51 : 0.84 with a water/(OPC + pfa) ratio of 0.4. Three types of steel fibres were used, namely melt extract (ME), corrosion resistant (CR) and low-carbon steel fibres (MS). The latter two fibres

TABLE I Details of fibres

Mix	Fibre type	Fibre details				
		l (mm)	d (mm)	$\frac{l}{d}$	v_f (%)	$v_f \frac{l}{d}$
A	Melt extract (ME)	25.0	0.51	49	3.0	147
	Low-carbon steel (MS)	28.2	0.48	60	2.5	147
	Corrosion resistant (CR)	40.0	0.60	66	2.2	145
B	Melt extract (ME)	26.5	0.44	60	1.7	100
	Low-carbon steel (MS)	28.2	0.48	60	1.7	100
	Corrosion resistant (CR)	40.0	0.60	66	1.7	112

had hooked ends. Details of fibre dimensions and volume fractions used are given in Table I.

Prism specimens of dimensions 100 mm × 100 mm × 500 mm were manufactured and these were exposed to different curing conditions which included sea-water spray cycles in the laboratory simulating splash zone exposure (Sh), laboratory air curing (LbA), laboratory water curing (LbW), and curing in the tidal zone at Aberdeen beach (Bh). Flexural tests were conducted after 150, 300 and 1200 marine cycles of exposure for both Mixes A and B and in addition at 2000 cycles on specimens of Mix A only. The specimens were tested under four-point bending and an automatic plot of the load–deflection curves was recorded. The point of deviation from linearity of the load–deflection graph was taken as the first crack strength.

4.2. Test results and discussion

4.2.1. Ultimate flexural strength

In order to obtain an equation for the ultimate flexural strength, a regression analysis was carried out using Equation 21, between $\sigma_{cu}/[v_f(l/d)]$ and $\sigma_m(1 - \beta v_f)/[v_f(l/d)]$ where the orientation factor β was taken as 0.41 [12]. The slope of the resulting linear relationship gives the value of constant $1.5K$ in Equation 21 and the intercept represents the value of the constant parameter ($2.44\tau\beta$). The modulus of rupture values, at different ages, of the OPC/pfa mixes (Mix B) incorporating different steel fibres yielded the following regression equation

$$\sigma_{cu} = 0.63\sigma_m(1 - \beta v_f) + 4.72 v_f \frac{l}{d} \qquad (24)$$

with a coefficient of correlation of 0.91. A straight line representing the above expression, together with the experimental points is plotted in Fig. 7.

The flexural strength Equation 23 which is based on previously available data [6] is also plotted in Fig. 7, and evidently does not represent the experimental results of the OPC/pfa mixes of sfrc in this figure. This is likely to be because Equation 23 was derived by regression analysis on a wide range of experimental data based predominantly on sfrc mixes made with OPC and without pozzolanic additives such as pfa.

Fig. 8 represents the flexural strength results of ordinary Portland cement mixes (Mix A) of this investigation, which were reinforced with ME, MS and CR fibres. In this figure, Equation 23 is valid for the experimental results whereas Equation 24 is unrepresentative. This must be because Equation 23 has been obtained for OPC-based mixes of sfrc.

The most likely reason for the different equations being valid to composites with and without pfa is the influence of the matrix on bond strength, τ. The inclusion of pfa in the matrix appears to affect τ significantly. This was also observed in another investigation of the free shrinkage of sfrc where the coefficient of friction, μ, at the fibre–matrix interface was found to be significantly different for mixes incorporating pfa [13, 14].

4.2.2. First crack flexural strength

By making suitable assumptions with respect to stress distribution and position of neutral axis at first crack, a similar expression for first crack flexural strength,

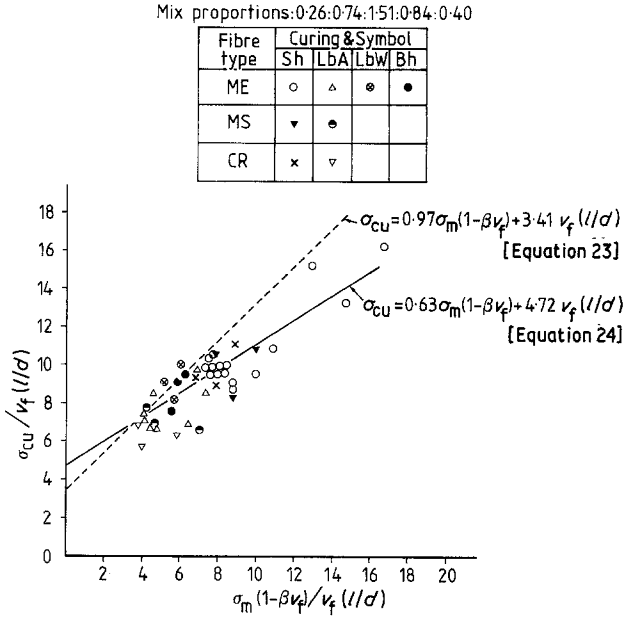

Figure 7 Ultimate flexural strength results of Mix B reinforced with steel fibres.

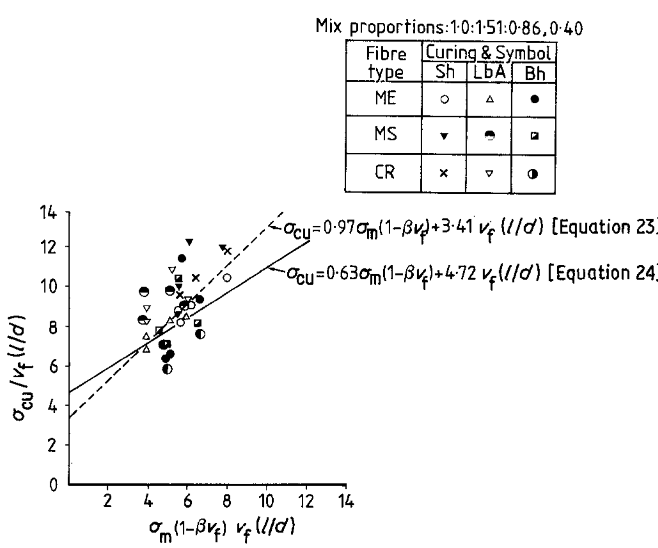

Figure 8 Ultimate flexural strength results of Mix A reinforced with steel fibres.

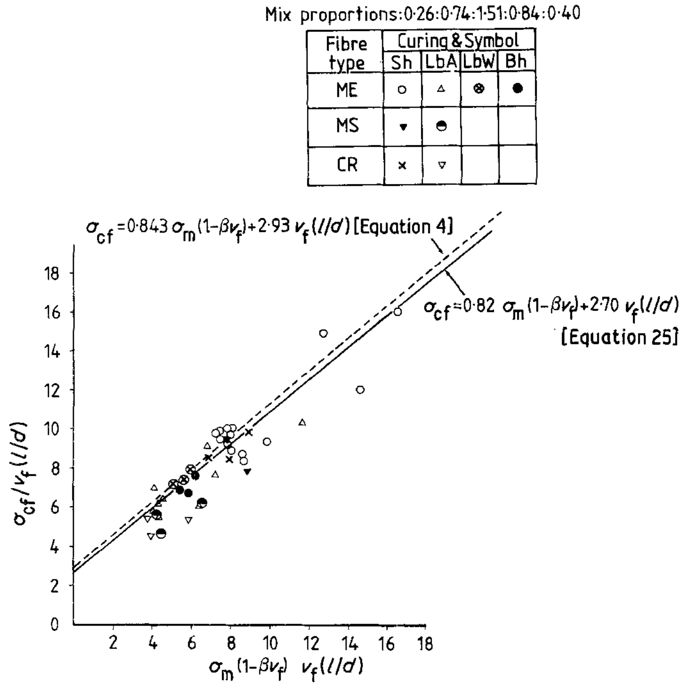

Figure 9 First crack flexural strength results of Mix B reinforced with steel fibres.

σ_{cf}, can be derived for sfrc, and the values of the constant factors may be derived by regression analysis of experimental data. This has been carried out on the data for Mix B specimens reinforced with different steel fibres and gives the following relationship with a coefficient of correlation of 0.97

$$\sigma_{cf} = 0.82\sigma_m(1 - \beta v_f) + 2.70 v_f \frac{l}{d} \qquad (25)$$

where σ_m is the modulus of rupture of the matrix. A line representing the above expression together with the experimental points is plotted in Fig. 9. The first crack flexural strength equation derived by Swamy and Mangat [6] is also plotted in Fig. 9 which practically coincides with the line representing Equation 25.

First crack flexural strength data for OPC mixes (Mix A) reinforced with ME, MS and CR fibres are plotted in Fig. 10. Equation 25 shows excellent correlation with the experimental results.

4.2.3. *Verification of theory*

In Section 2.1 it was stated that the theoretical Equations 1 and 2 derived by Hannant [3] for the flexural and tensile strength of sfrc were unsatisfactory because of the exclusion of the matrix strength component in the composite equations. This was verified by plotting the τ values at different $v_f(l/d)$ ratios in Figs 2 and 3, which showed unacceptable increases in τ with decreasing $v_f(l/d)$. The same test is now applied to the composite strength equations derived in this paper in order to confirm that the matrix strength component is a constituent of such expressions. Using a wide range of flexural strength data obtained from Mangat [7], values of τ have been calculated from Equation 21 assuming K as 0.65 and β as 0.41. These values of τ are plotted against $v_f(l/d)$ in Fig. 2. It is evident that $v_f(l/d)$ has no significant effect on these τ values, the bulk of which range between 1.8 and 3.4 N mm^{-2}. The scatter in these values is caused not by variations in $v_f(l/d)$ but by scatter of experimental data on flexural strength.

The above band of τ values is within the range 2.00 to 3.74 N mm^{-2} which was obtained from the law of mixtures for a wider range of mixes [15]. These values were obtained by substituting flexural strength information in the mixtures Equation 3. If, instead, the flexural strength Equation 21 was used, it would give τ values ranging between 1.64 and 3.1 N mm^{-2}. These values are similar to the range 1.8 to 3.4 N mm^{-2} obtained in this paper.

The principles used in the derivation of flexural strength expressions in this paper can be equally applied to the case of direct tensile stress and result in the following expression

$$\sigma_{ct} = \sigma_{mt}(1 - \beta v_f) + \beta\tau v_f \frac{l}{d} \qquad (26)$$

where σ_{ct} is the tensile strength of sfrc and σ_{mt} is the tensile strength of the matrix. Assuming $\beta = 0.41$ [12] and using a wide range of direct tensile strength data obtained from Johnston and Coleman [8], values of τ have been calculated from Equation 26. These values

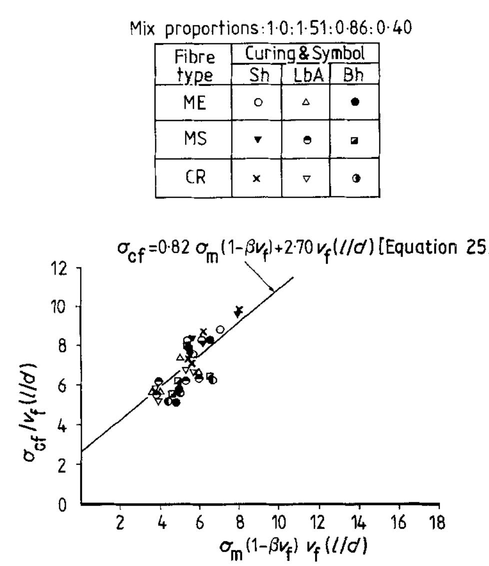

Figure 10 First crack flexural strength results of Mix A reinforced with steel fibres.

are plotted against $v_f(l/d)$ in Fig. 3. It is again clear that τ is independent of $v_f(l/d)$ thus confirming that any increase in τ with decreasing $v_f(l/d)$ is due to the omission of the matrix strength component in the composite equation.

5. Conclusions

At ultimate stress, the failure mechanism of sfrc in direct or flexural tension is essentially due to simultaneous matrix cracking and composite failure. Consequently, in any expression for composite strength, the matrix strength component should be a constituent of the equation.

The flexural strength of sfrc can be predicted from a composite equation of the form

$$\sigma_{cu} = A\sigma_m(1 - \beta v_f) + Bv_f\frac{l}{d}$$

where constants A and B account for the shift in neutral axis at ultimate stress, fibre–matrix interfacial bond strength and orientation of fibres. Because of the difficulty in precisely determining these factors, A and B are best derived empirically using experimental data. A similar expression can be derived for the direct tensile strength of such composites.

The values of A and B remain fairly constant for a very wide range of sfrc mixes manufactured with ordinary Portland cement and cured under different conditions. These values, however, are different for mixes incorporating pulverised fuel ash.

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